Problem 1

a) $f(x) = 2x^3 + 3x^2 - 12x$

By differentiating

$f'(x) = 6x^2 + 6x - 12$

To find critical points $f'(x) = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x+2) - 1(x+2) = 0$

$\Rightarrow (x-1)(x+2) = 0$

Critical Points are $x = 1, -2$

$\Rightarrow$ Sign chart of our function

Interval	Test x-value	$f'(x)$	Conclusion
$(-\infty, -2)$	$x = -3$	$f'(-3) = 24 > 0$	f is increasing ↑
$(-2, 1)$	$x = 0$	$f'(0) = -12 < 0$	f is decreasing ↓
$(1, \infty)$	$x = 2$	$f'(2) = 24 > 0$	f is increasing ↑

b) $f(x) = (x^2-4)^2$

By differentiating

$$f'(x) = 4x(x^2-4)$$

To find Critical Points $f'(x) = 0$

$$\Rightarrow 4x(x^2-4) = 0$$

$$\Rightarrow 4x(x+2)(x-2) = 0$$

Critical Points are $x = 0, 2, -2$

$\Rightarrow$ Sign Chart of our function is

Interval	Test x-Value	$f'(x)$	Conclusion
$(-\infty, -2)$	$x = -3$	$f'(-3) = -60 < 0$	f is decreasing ↓
$(-2, 0)$	$x = -1$	$f'(-1) = 12 > 0$	f is increasing ↑
$(0, 2)$	$x = 1$	$f'(1) = -12 < 0$	f is decreasing ↓
$(2, \infty)$	$x = 3$	$f'(3) = 60 > 0$	f is increasing ↑

Problem 2

a) Number of Critical Points $f(x)$ have on $[0, 8]$

A Critical Point of a continuous function f is a point at which the derivative is zero or undefined.

From graph Critical Points are $(3, 3)$, $(5, 5)$, $(7, 1)$

b) Maximum value from graph on $[0, 8]$ is

at $x = 0 \Rightarrow f(0) = 6$

Maximum value $= f(0) = 6$

c) Local Maxima values are

$(5, 5)$

d) Closed interval on which both the minimum and maximum values of $f(x)$ occur at Critical Points

$[0, 7]$

Problem 5

a) Critical Points occur at $f'(x) = 0$ and Changes sign

From graph Critical Points are F

$f'(x) > 0$ increasing

$f'(x) < 0$ decreasing

From graph (A, F) $f(x)$ is decreasing

(F, J) $f(x)$ is increasing

b)

Point of inflection $\Rightarrow f'(x)$ has extreme value

From graph Point of inflection are B, D, G, I

$f'(x)$ is increasing $\Rightarrow f(x)$ is Concave up

From graph (D, G) ; (I, J) is Concave up.

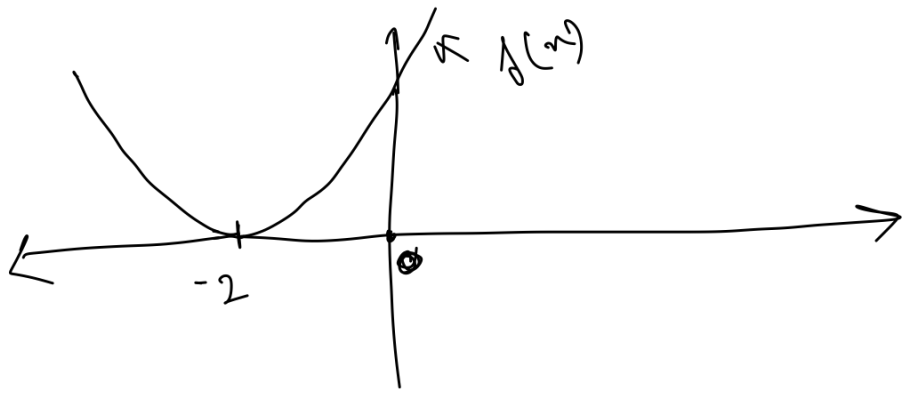
Problem 6

$f' < 0$ at $x < -2 \Rightarrow$ decreasing $(-\infty, -2)$

$f' > 0$ at $x > -2 \Rightarrow$ increasing $(-2, \infty)$

$f'' > 0$ at $x < 0 \Rightarrow$ Concave up

$f'' < 0$ at $x > 0 \Rightarrow$ Concave down



Problem 7 Point of inflection occurs at $f''(x) = 0$

$f(x) = 3x^5 - 30x^4 + 9x^3 + 45x^2 + 60x + 1$

$f'(x) = 15x^4 - 120x^3 + 39x^2 + 25x + 60$

$f''(x) = 60x^3 - 360x^2 - 6ax + 2b$

We are given inflection Points are $x = 2, 3$

$\Rightarrow f''(2) = 0 \; ; \; f''(3) = 0$

$$f'(x) = 60x^3 - 360x^2 - 6ax + 2b$$

$$f''(2) = 60(2)^3 - 360(2)^2 - 6a(2) + 2b = 0$$

$$\Rightarrow -12a + 2y - 960 = 0 \quad \text{— (1)}$$

Similarly,

$$f''(3) = 60(3)^3 - 360(3)^2 - 6a(3) + 2b = 0$$

$$\Rightarrow -18a + 2b - 1620 = 0 \quad \text{— (2)}$$

Solving (1), (2)

$$a = -110, \; b = -180$$